Sample Question Paper 2020-21

Max. Marks: 80 | **Duration:3 hours**

General Instructions:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:

1. It consists of two sections- I and II
2. Section I has 16 questions. Internal choice is provided in 5 questions.
3. Section II has four case study-based questions. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

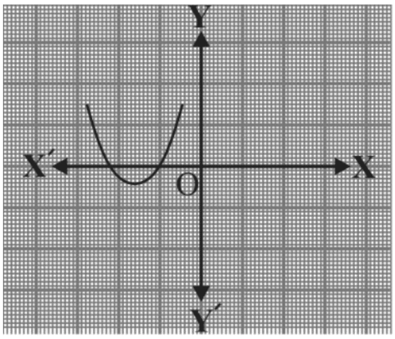
Part – B:

1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

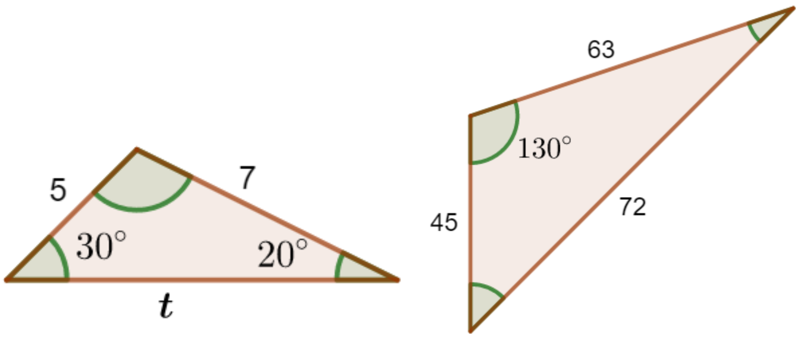
PART – A

SECTION – I

1. If the sum of the roots of the equation $3x^2$-(3k-2) x-(k-6) = 0 is equal to the product of its roots, then calculate k?
2. In the given figure, the graph of the polynomial p(x) is shown. The number of zeros of p(x) is ______ .



3. In a cyclic quadrilateral ABCD, ∠A = (2x + 4)°, ∠B = (y + 3)°, ∠C = (2y + 10)° and ∠D = (4x – 5)°, the find the value of ∠A.
4. In the figure, what is the value of t?



OR

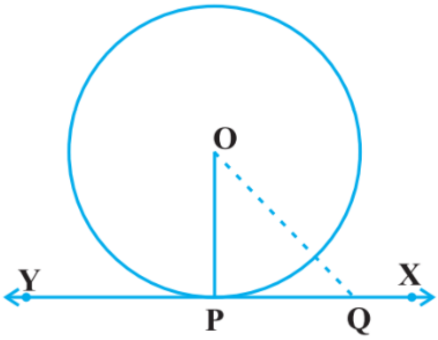
If ABC is an isosceles triangle, right angled at C. Prove that $AB^2 = 2AC^2$.

5. If secθ $=\frac{1}{x}$, find the value of sinθ in terms of x.

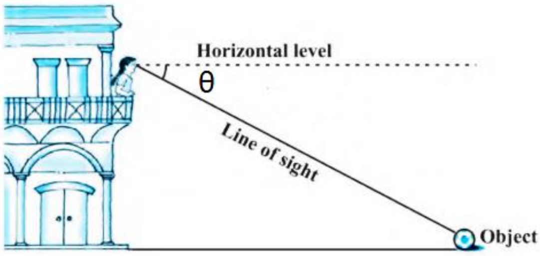
OR

Find the value of $\sin^2 30° - \cos^2 60° + tan45° - cosec45°$.

6. If α and β are the roots of equation $x^2 + ax + b = 0$, then a + b equals _______ .
7. In the following figure, if OQ = 13 cm and PQ = 12 cm, then calculate the radius of circle.



8. The next term of AP -4, 1, 6, … is
9. Find the centroid of the triangle whose vertices are (1, 4), (-1, -1), (3, -2).
10. The θ in the following figure is



11. If first term of an AP is 5 and the 10^{th} term is 45, then sum of first ten terms of the AP is ______.

OR

In an A.P the first term is equal to twice of the fourth term of that A.P, then d : a = ______, where 'd' is common difference and 'a' is first term.

12. If cos θ = 0.6, then 5sinθ − 3tanθ = ______.
13. From a point P, a tangent is drawn to a circle, and it meets the circle at point T. If the distance of the centre of the circle from P is 13cm and the radius of the circle is 12cm. Find the length PT.
14. If LCM(a, b) = 210 and HCF(a, b) = 6 then a × b = ________.

OR

Find whether the rational number $\frac{23}{2500}$ will have a terminating decimal expansion or a non-terminating decimal expansion.

15. If the total surface area of a solid hemisphere is 462 cm^2, its radius is ______.
16. For what value of k does the following equation have real and equal roots?

$2x^2 - (2k + 1)x + k = 0$

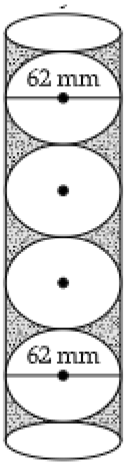
OR

If x = 1, is a root of the equation, $f(x) = 2kx^2 - x + k$. Then, find the value of k.

Section-II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark

17. Five tennis balls, diameter 62 mm are placed in cylindrical card tubes (as shown in figure)



(i) Find the radius of the tennis balls

(a) 30 mm

(b) 29 mm

(c) 31 mm

(d) 32 mm

(ii) Volume of 1 ball is equal to

(a) 125 cm^3

(b) 123.5 cm^3

(c) 120.30 cm^3

(d) 124.84 cm^3

(iii) Find the height of the tube

(a) 300 mm

(b) 320 mm

(c) 310 mm

(d) 301 mm

(iv) Find the volume of the tube

(a) 963 cm^3

(b) 966.3 cm^3

(c) 939.23 cm^3

(d) 936.29 cm^3

(v) Find the volume of unfilled space (shaded area) in the tube.

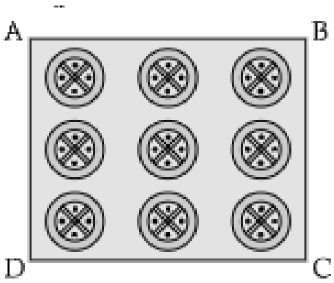
(a) 310.9 cm^3

(b) 312.09 cm^3

(c) 301.90 cm^3

(d) 321.09 cm^3

18. In a school, a Design exam is conducted in Class X. Rubina wins 1st prize. She made a square embroider handkerchief with 9 circular thread designs on it.



(i) On a square handkerchief, nine circular designs each of radius 7 cm are made (see in figure). Find the circumference of one of the circular design.

(a) 41 cm

(b) 42 cm

(c) 43 cm

(d) 44 cm

(ii) Find the total area of 9 circles if radius of each circle is 7 cm.

(a) 1380 cm^2

(b) 1385 cm^2

(c) 1386 cm^2

(d) 1384 cm^2

(iii) The area of circle having 'r' is equal to

(a) πr^2

(b) πr^3

(c) $2\pi r^2$

(d) $3\pi r^2$

(iv) If radius of circle is 4r, the area of circle is equal to

(a) $50.20\ r^2$ sq. units

(b) $50.28\ r^2$ sq. units

(c) $51.24\ r^2$ sq. units

(d) $52.24\ r^2$ sq. units

(v) Area of square is equal to

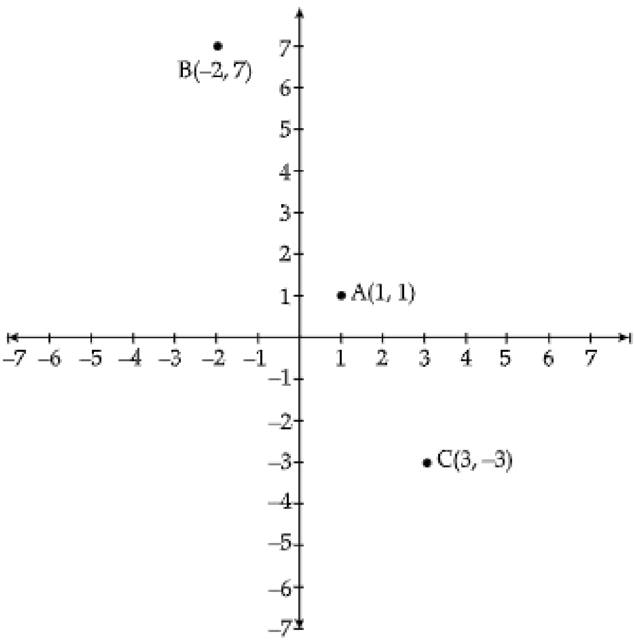
(a) 4a

(b) a^2

(c) 2a

(d) 3a

19. The given figure shows the arrangement of chairs in a classroom. Dinesh, Mohan and Sohan are seated at A(1, 1), B(–2, 7) and C(3, –3) respectively



(i) Find the distance between Dinesh and Sohan.

(a) $2\sqrt{5}$ units

(b) $2\sqrt{3}$ units

(c) $2\sqrt{7}$ units

(d) $3\sqrt{2}$ units

(ii) Find the distance between Dinesh and Mohan.

(a) $5\sqrt{3}$ units

(b) $5\sqrt{2}$ units

(c) $3\sqrt{5}$ units

(d) $2\sqrt{5}$ units

(iii) Name the quadrant in which Sohan is seated.

(a) I quadrant

(b) II quadrant

(c) III quadrant

(d) IV quadrant

(iv) Name the quadrant in which Dinesh is seated.

(a) I quadrant

(b) II quadrant

(c) III quadrant

(d) IV quadrant

(v) Which of the following is the correct distance formula.

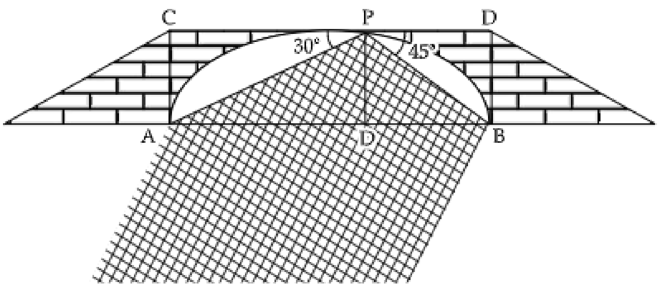
(a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(b) $[(x_1 - x_2) + (y_1 - y_2)]^2$

(c) $(x_1 + x_2)^2 + (y_1 - y_2)^2$

(d) $(x_1 - x_2) - (y_1 + y_2)^2$

20. From a point on a bridge across a river, the angle of depression of the banks on opposite sides of the river 30° and 45°, respectively



(i) If the bridge is at a height of 3 m from the banks, find the width of the river

(a) $2(\sqrt{3} + 1)$ m

(b) $(\sqrt{3} + 1)$ m

(c) $(\sqrt{3} + \quad 2)$ m

(d) $3(\sqrt{3} + 1)$ m

(ii) Name the ΔAPD.

(a) Acute Angled triangle

(b) Right Angled triangle

(c) Obtuse Angled triangle

(d) Equilateral triangle

(iii) In ΔAPD, $\tan 30^\circ = ?$

(a) AD/DP

(b) AP/AD

(c) PD/AD

(d) AD/AP

(iv) The value of $\tan 45^\circ$ is equal to

(a) 0

(b) 2

(c) 1

(d) $1/\sqrt{3}$

(v) The value of tangent in right angle triangle is equal to

(a) $\frac{\text{Perpendicular}}{\text{Base}}$

(b) $\frac{\text{Base}}{\text{Perpendicular}}$

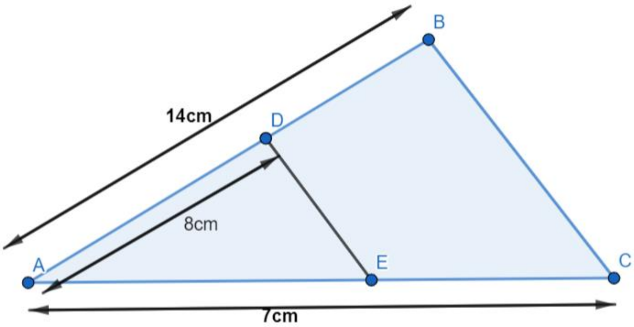
(c) $\frac{\text{Hypotenuse}}{\text{Base}}$

(d) $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

21. In figure, DE || AC, and AB = 14cm, AD = 8cm, and BC = 7cm. Find the value of BE.



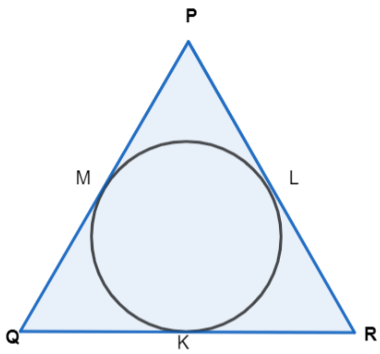
22. Using Euclid's division algorithm, find the HCF of 84, 144 and 400.
23. Find the sum of first 100 odd integers.

OR

If a_n is the nth term and d is the common difference of an A.P. Prove that:

$a_n - a_{n-1} = d$

24. In the figure below, the incircle of ΔPQR touches the sides QR, RP and PQ at K, L and M respectively. If PQ = PR, prove that KQ = KR



25. Five years hence, the age of Anjali will be three times that of Dimple. Five years ago, Anjali's age was seven times that of Dimple. What are their present ages?

OR

Solve the following equations and find the value of k, where $k = (y - x)^3$.

$2x - y = 11$

$x + y = 25$

26. In an equilateral triangle of side 'a'cm and height of one of its altitude as 'h'cm, prove that $a = \frac{2}{\sqrt{3}}h$.

Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

27. Prove that $(\sqrt{7} + \sqrt{11})$ is irrational.

OR

Prove that if a and b are odd positive integers, then $a^2 + b^2$ is even but not divisible by 4.

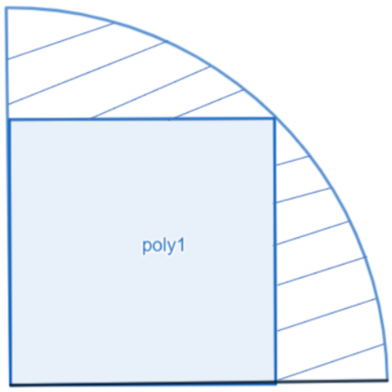
28. If (-5, 3), (-7, -2), (x, 2) and (2, y) are the vertices of a parallelogram taken in order. Find the values of x and y.

29. Draw a circle of radius 2 cm. Take two points P and Q on one of its extended diameter each at a distance of 4 cm from its centre. Draw tangents to the circle form the two points P and Q.

30. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if two zeroes are 1 and 1/2.

31. A square is inscribed in a quadrant of a circle whose radius is 28cm as shown in the figure below. Find the area of the shaded region.

$$\left[Take\ \pi = \frac{22}{7}\right]$$



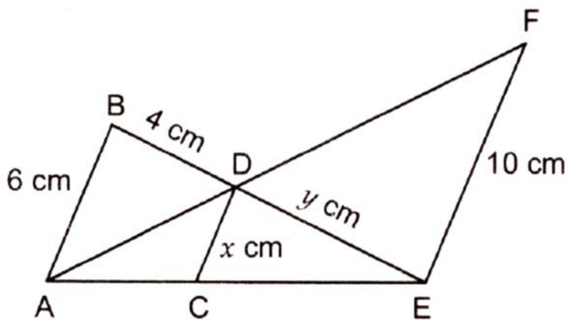
OR

The length of the minute hand of a clock is 10cm. Find the area swept by the minute hand of a clock from the time 6:10 to 7:05.

32. Find the roots of the equation:

$$\frac{1}{x} - \frac{1}{x-3} = \frac{4}{3}; x \neq 0, 3$$

33. In fig., we have AB∥CD∥EF. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm and DE = y cm, calculate the values of x and y.



Part –B

All questions are compulsory. In case of internal choices, attempt anyone.

34. A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm which are filled completely. Find the diameter of the cylindrical vessel.

35. The following is the distribution of height of students of a certain class in a certain city.

Height (in cms):	160-162	163-165	166-168	169-171	172-174
No. of students:	15	118	142	127	18

Find the median height.

36. From a point 200m above the lake, the angle of elevation of a stationary helicopter is 30°, and the angle of depression of reflection of the helicopter in the lake is 45°. Find the height of the helicopter.

OR

If the angles of elevation of a tower from two points at distances a and b, where a > b from its foot and in the same straight line from it are 30° and 60° respectively, the find the value of $\sqrt{\frac{a}{b}}$.

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Hints & Solutions

PART – A

SECTION – I

1. Solution: Let the roots of the given quadratic equation $3x^2-(3k-2)x-(k-6)=0$ be α and β.

   Now,

   sum of roots = $\alpha+\beta = (3k-2)/3$ and,

   product of roots = $\alpha\beta = -(k-6)/3$

   [$\because$ If α and β are the roots of quadratic equation $ax^2+bx+c=0$ then $\alpha+\beta = -b/a$ and $\alpha\beta = c/a$]

   According to question-

   sum of roots = product of roots

   $\therefore \alpha+\beta = \alpha\beta$

   $\Rightarrow (3k-2)/3 = -(k-6)/3$

   $\Rightarrow 3k-2 = -k+6$

   $\Rightarrow 4k = 8$

   $\therefore k = 2$

   Hence, the value of k is 2.

2. Solution: The zeroes of polynomial means that value of polynomial becomes zero.

   In the above graph, the curve depicts the polynomial and it gets zero at two points, therefore p(x) has two zeroes.

3. Solution: We know that, In a cyclic quadratic opposite angles are supplementary.

   $\angle A + \angle C = 180°$

   $\Rightarrow 2x + 4 + 2y + 10 = 180$

   $\Rightarrow 2(x + y) = 166$

   $\Rightarrow x + y = 83$

   $\Rightarrow y = 83 - x$ [1]

   Also, $\angle B + \angle D = 180°$

   $\Rightarrow y + 3 + 4x - 5 = 180$

   $\Rightarrow 4x + y = 182$

   $\Rightarrow 4x + 83 - x = 182$ [From 1]

   $\Rightarrow 3x = 99$

   $\Rightarrow x = 33°$

   $\Rightarrow \angle A = 2(33) + 4 = 70°$

4. Solution: In ΔABC and ΔA′B′C′

$\frac{AB}{A'B'} = \frac{5}{45} = \frac{1}{9}$

$\frac{AC}{A'C'} = \frac{7}{63} = \frac{1}{9}$

Also, In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 30° + 20° = 180°$

$\Rightarrow \angle A = 130°$

So here, $\frac{AB}{A'B'} = \frac{AC}{A'C'}$

$\angle A = \angle A'$

$\Rightarrow \Delta ABC \sim \Delta A'B'C'$ [By SAS Similarity]

$\Rightarrow \frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$

$\Rightarrow \frac{1}{9} = \frac{t}{72}$

$\Rightarrow t = 8$ units

OR

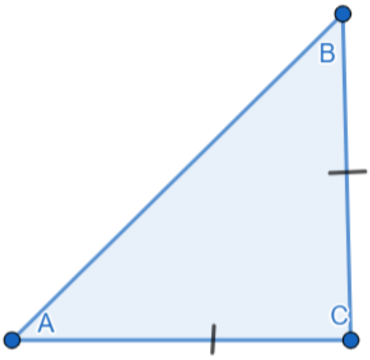
Solution:

By Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$AB^2 = AC^2 + BC^2$

[Given: AC = BC, as it is an isosceles triangle]



$AB^2 = 2AC^2$

Hence, Proved.

5. Solution: We know that,

$\sec\theta = \frac{1}{\cos\theta}$

Therefore, $\cos\theta = x$

Now,

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + x^2 = 1$

$\sin^2\theta = 1 - x^2$

$\sin\theta = \sqrt{1-x^2}$

OR

Solution: To Find: $\sin^2 30° - \cos^2 60° + tan45° - cosec45°$

We know that,

$$sin30° = \frac{1}{2}, cos60° = \frac{1}{2}, tan45° = 1 \ and \ cosec45° = \sqrt{2}$$

Putting the values, we get,

$$\sin^2 30° - \cos^2 60° + tan45° - cosec45° = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1 - \sqrt{2}$$

$$\sin^2 30° - \cos^2 60° + tan45° - cosec45° = 1 - \sqrt{2}.$$

6. Solution: For a quadratic equation, we know that

$$\text{sum of roots} = -\frac{\text{coefficient of x}}{\text{coefficient of x}^2}$$

$$\Rightarrow \alpha + \beta = -\frac{a}{1}$$

$\Rightarrow -a = \alpha + \beta$[1]

$$\text{Product of roots} = \frac{\text{constant term}}{\text{coefficient of x}^2}$$

$$\Rightarrow \alpha\beta = \frac{b}{1}$$

$\Rightarrow b = \alpha\beta$ [2]

From [1] and [2], we have

$\Rightarrow b + a = \alpha\beta - \alpha - \beta$

7. Solution: We know that, tangent through a circle is perpendicular to the radius through point of contact.

$\Rightarrow OP \perp XY$ and ΔOPQ is a right-angled triangle

$\Rightarrow OP^2 + PQ^2 = OQ^2$

$\Rightarrow r^2 + 12^2 = 13^2$ [$\because$ OP = radius (r)]

$\Rightarrow r^2 = 169 - 144 = 25$ $\Rightarrow r = 5$ cm

Hence, Option B is correct.

8. Solution: We know that, nth term of an AP is

$a_n = a + (n - 1)d$

where, a = first term and d = common difference

Here,

First term, a = -4

Common Difference, $d = a_2 - a_1 = 1 - (-4) = 5$

To find: 4[th] term i.e. a_3

$\Rightarrow a_3 = a + 3d = -4 + 3(5) = 11$

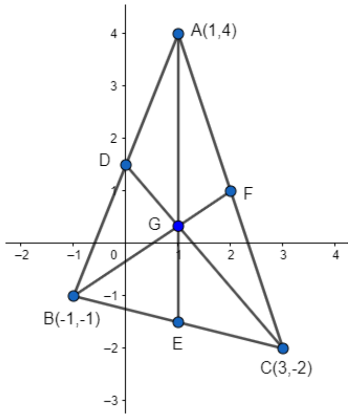
9. Solution: Given: Points vertices (1, 4), (-1, -1), (3, -2)

To find: The centroid of triangle.

Formula Used:

Centroid of a triangle for (x_1, y_1), (x_2, y_2) and (x_3 , y_3) is

$$G\ (x, y)\ =\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$



We know that centroid of a triangle for (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$G(x, y)\ =\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

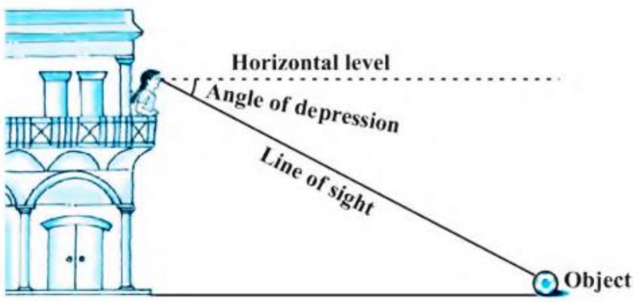
∴For coordinates (1, 4), (-1, -1), (3, -2),

Centroid of triangle = $\left(\frac{1-1+3}{3}, \frac{4-1-2}{3}\right)$

= $\left(1, \frac{1}{3}\right)$

Hence, centroid of triangle is $\left(1, \frac{1}{3}\right)$

10. Solution: The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed.



11. Solution: We know, sum of 'n' terms of an AP

$$S_n = \frac{n}{2}(a + a_n)$$

Where a and a_n are first and last term respectively.

Here, a = 5

n = 10

a_n = 45

Putting values, we get

$$S_{10} = \frac{10}{2}(45 + 5) = 250$$

OR

Solution: We know that nth term of an A.P is given by,

$a_n = a + (n - 1)d$

where, a = first term, n = number of terms, and d = common difference.

Therefore,

First term = a

Fourth term, $a_4 = a + (4 - 1)d$

$a_4 = a + 3d$

According to question,

a = 2(a + 3d)

a = 2a + 6d

a = -6d

$$\frac{d}{a} = -\frac{1}{6}$$

12. Solution: Given cos θ = 0.6

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

⇒ sin θ = 0.8

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \tan\theta = \frac{4}{3}$$

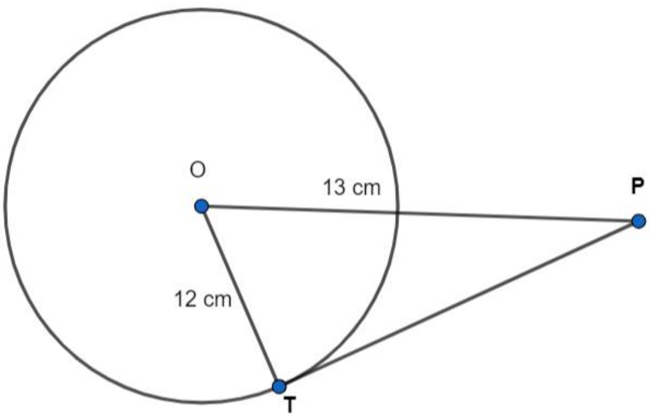
According to the question, the required problem needs us to find

5 sin θ- 3 tan θ

$$= 5 \times 0.8 - 3 \times \frac{4}{3}$$

= 4 – 4 = 0

13. Solution:



Theorem: A tangent to a circle is perpendicular to the radius through the point of contact.

$\angle PTO = 90^\circ$

Now, applying Pythagoras theorem in ΔPTO,

$OP^2 = OT^2 + PT^2$

$13^2 = 12^2 + PT^2$

$PT^2 = 169 - 144$

$PT^2 = 25$

PT = 5cm

14. Solution: We know that, LCM(a, b) × HCF(a, b) = a × b

$\Rightarrow 210 \times 6 = ab$

$\Rightarrow ab = 1260$

OR

Solution: We know that, for a rational number $\frac{p}{q}$ to have a terminating decimal expansion, q should be of the form $= 2^a \times 5^b$

Therefore, in a given number $\frac{23}{2500}$, denominator 2500 can be factorised as,

$2500 = 2 \times 2 \times 5 \times 5 \times 5 \times 5$

$2500 = 2^2 \times 5^4$

As 2500 can be expressed as $2^2 \times 5^4$, it will have a terminating decimal expansion.

15. Solution: Let the radius of solid sphere be r cm

Total surface area of solid hemisphere $= 3\pi r^2$

Given, total surface area of solid hemisphere $= 462\ cm^2$

$\therefore \quad 3\pi r^2 = 462\ cm^2$

$\Rightarrow \quad 3 \times 22/7 \times r^2 = 462\ cm^2$

$\Rightarrow \quad r^2 = 462 \times 1/3 \times 7/22\ cm^2 = 49\ cm^2$

$\Rightarrow \quad r = 7$ cm

16. Solution: We know that, in a quadratic equation, $ax^2 + bx + c = 0$.

For real and equal roots, $b^2 - 4ac = 0$

Given Equation: $2x^2 - (2k + 1)x + k = 0$

So, we have, a = 2, b = -(2k + 1), and c = k

Therefore,

$\{-(2k + 1)\}^2 - 4 \times 2 \times k = 0$

$4k^2 + 1 + 4k - 8k = 0$

$4k^2 - 4k + 1 = 0$

This is a formula for $(2k - 1)^2$, so,

$(2k - 1)^2 = 0$

$2k - 1 = 0$

$k = ½$

Hence, for k = ½ the given quadratic equation has real and equal roots.

OR

Solution:

Given: $f(x) = 2kx^2 - x + k$

If x = 1, is a root of the equation, it will satisfy the equation. Therefore,

$f(1) = 0$

$2k(1)^2 - 1 + k = 0$

$2k - 1 + k = 0$

$3k - 1 = 0$

$k = 1/3$

Section-II

17. (i) Answer: 31 mm
    (ii) Answer: 124.84 cm^3
    (iii) Answer: 310 mm
    (iv) Answer: 936.29 cm^3
    (v) Answer: 321.09 cm^3
18. (i) Answer: 44 cm
    (ii) Answer: 1386 cm^2
    (iii) Answer: πr^2
    (iv) Answer: 50.28 r^2 sq. units
    (v) Answer: a^2
19. (i) Answer: $2\sqrt{5}$ units
    (ii) Answer: $3\sqrt{5}$ units
    (iii) Answer: IV quadrant
    (iv) Answer: I quadrant
    (v) Answer: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
20. (i) Answer: $3(\sqrt{3} + 1)$ m
    (ii) Answer: Right Angled triangle
    (iii) Answer: PD/AD
    (iv) Answer: 1
    (v) Answer: $\frac{\text{Perpendicular}}{\text{Base}}$

Part –B

21. Solution:

From the figure,

AB = AD + DB

14cm = 8cm + BD

BD = 6cm

Now, let EC = xcm

Then, AE + EC = 7cm

AE + x = 7cm

AE = (7 – x)cm

Given: DE || BC, therefore by basic proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{8}{6} = \frac{7 - x}{x}$$

8x = 6(7 – x)

8x = 42 – 6x

14x = 42

$x = \frac{42}{14}$

x = 3cm

AE = (7 – 3)cm = 4cm

Hence, the value of AE is 4cm.

22. Solution:

First using Euclid's division algorithm for 84 and 144

We get, 144 = 84 × 1 + 60

84 = 60 × 1 + 24

60 = 24 × 2 + 12

24 = 12 × 2 + 0

Now, the remainder is zero, HCF of 84 and 144 is 12.

Now, 400 = 12 × 33 + 4

12 = 4 × 3 + 0

Now, remainder is zero, HCF of 400 and 12 is 4.

Hence, the HCF of 84, 144, and 400 is 4.

23. Solution: The progression for odd numbers looks like 1, 3, 5, 7, 9, ……..

Now, for this A.P

First term, a = 1

Common difference, d = 3 – 1 = 2

Number of terms = 100

We know that the sum of n terms is given by,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore,

$S_{100} = \frac{100}{2}[2 \times 1 + (100 - 1) \times 2]$

$S_{100} = 50[2 + 198]$

$S_{100} = 50 \times 200$

$S_{100} = 10000$

OR

Solution:

Let a be the first term, of an A.P with a_n as the nth term and d as the common difference.

We know that,

$a_n = a + (n - 1)d$

Similarly,

$a_{n-1} = a + (n - 1 - 1)d = a + (n - 2)d$

$a_n - a_{n-1} = a + (n - 1)d - [a + (n - 2)d]$

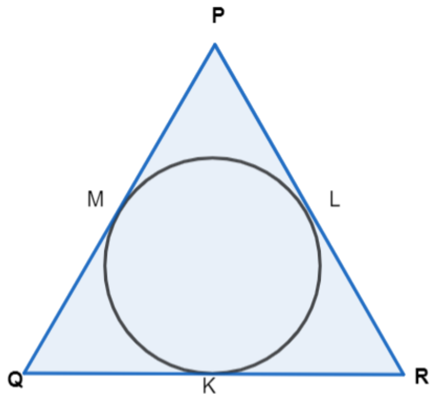
$a_n - a_{n-1} = a + nd - d - a - nd + 2d$

$a_n - a_{n-1} = 2d - d$

$a_n - a_{n-1} = d$

Hence, Proved.

24. Solution:



Theorem: The lengths of the tangents drawn from an external point to a circle are equal.

Therefore, from the above figure we have,

PM = PL

MQ = KQ

KR = LR

Adding them we get,

PM + MQ + KR = PL + KQ + LR

PQ + KR = PR + KQ

[Since, PM + MQ = PQ and PL + LR = PR]
[Also, PQ = PR(Given)]
KR = KQ
Hence, Proved.

25. Solution:

Let the present age of Anjali be x and of Dimple be y.
Five years hence,
Age of Anjali = (x + 5) years
Age of Dimple = (y + 5) years
Age of Anjali = 3(Age of Dimple)
$x + 5 = 3(y + 5)$
$x + 5 = 3y + 15$
$x - 3y = 10$......(1)
Five years ago,
Age of Anjali = (x − 5)
Age of Dimple = (y − 5)
Age of Anjali = 7(Age of Dimple)
$x - 5 = 7(y - 5)$
$x - 5 = 7y - 35$
$x - 7y = -30$......(2)
Subtracting equation 2 from equation 1, we get,
$(x - 3y) - (x - 7y) = 10 - (-30)$
$x - 3y - x + 7y = 10 + 30$
$4y = 40$
y = 10 years
Putting the value of 'y' in equation 1, we get,
$x - 3 \times 10 = 10$
$x - 30 = 10$
x = 40 years
Hence, the age of Anjali is 40 years and age of Dimple is 10 years.

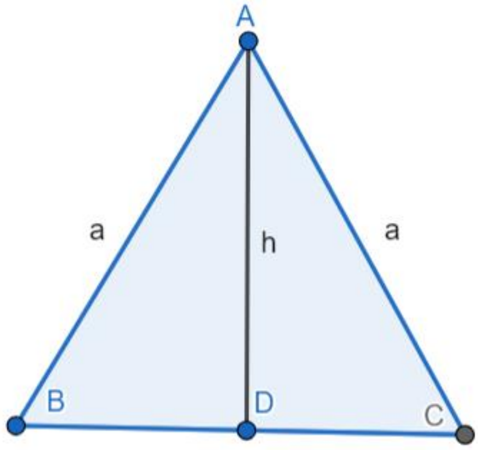
OR

Solution:

Let
$2x - y = 11$ .....(1)
$x + y = 25$....(2)
Multiplying equation 2 with 2, we get,
$2(x + y) = 2 \times 25$
$2x + 2y = 50$.....(3)
Subtracting equation 2 from equation 3, we get,

$(2x + 2y) - (2x - y) = 50 - 11$
$2x + 2y - 2x + y = 39$
$3y = 39$
$y = 13$
Putting the value of 'y' in equation (2), we get,
$x + 13 = 25$
$x = 12$
Therefore, $x = 12$ and $y = 13$
Now, $k = (13 - 12)^3 = (1)^3 = 1$
Hence, the value of k is 1

26. Solution:



Theorem: Altitude of an equilateral triangle bisects the corresponding side.
Therefore, BD = CD = ½ a
Now, in ΔADC,
AD = h cm
CD = ½ a cm
AC = a cm
Applying the Pythagoras theorem, we get,
$AC^2 = AD^2 + CD^2$
$a^2 = h^2 + (a/2)^2$

$$a^2 = h^2 + \frac{a^2}{4}$$
$$h^2 = a^2 - \frac{a^2}{4}$$
$$h^2 = \frac{4a^2 - a^2}{4}$$
$$h^2 = \frac{3a^2}{4}$$
$$h = \frac{\sqrt{3}}{2}a$$
$$a = \frac{2}{\sqrt{3}}h$$

Hence, Proved.

Part –B

27. Solution: We will prove this by contradiction. So, assuming the statement $(\sqrt{7} + \sqrt{11})$ is irrational to be true.

Let $k = \sqrt{7} + \sqrt{11}$

$\sqrt{11} = k - \sqrt{7}$

Squaring both sides, we get,

$11 = (k - \sqrt{7})^2$

$11 = k^2 + (\sqrt{7})^2 - 2 \times \sqrt{7} \times k$

$k^2 + 7 - 2\sqrt{7}k = 11$

$2\sqrt{7}k = 11 - 7 - k^2$

$2\sqrt{7}k = 4 - k^2$

$\sqrt{7} = \frac{4-k^2}{2k}$

As, k is a rational number, $\frac{4-k^2}{2k}$ is also n rational number.

But $\sqrt{7}$ is an irrational number.

So, there is a contradiction.

Therefore, statement is wrong.

Hence, $\sqrt{7} + \sqrt{11}$ is an irrational number.

OR

Solution: Any odd integer is of the form $(2q + 1)$ for some integer q.

Let $a = 2m + 1$ and $b = 2n + 1$

Therefore,

$a^2 + b^2 = (2m + 1)^2 + (2n + 1)^2$

$a^2 + b^2 = 4m^2 + 4m + 1 + 4n^2 + 4n + 1$

$a^2 + b^2 = 4\{(m^2 + n^2) + (m + n)\} + 2$

Let $k = \{(m^2 + n^2) + (m + n)\}$

Then,

$a^2 + b^2 = 4k + 2$

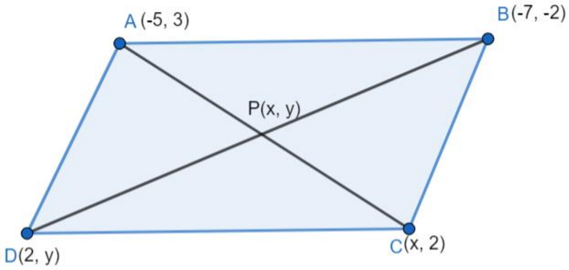
$a^2 + b^2 = 2(2k + 1)$

Therefore, $a^2 + b^2$ = even number.

As the multiple is of 2 and not 4. The number is not divisible by 4.

Hence, Proved.

28. Solution:



Given: Vertices of parallelogram A(-5, 3), B(-7, -2), C(x, 2) and D(2, y)

Theorem: Diagonals of a parallelogram bisect each other.

Therefore,

P(x, y) is the midpoint of BD and AC.

We know, by midpoint formula that midpoint of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Now,

For BD,

$(x, y) = \left(\frac{-7+2}{2}, \frac{-2+y}{2}\right)$

$(x, y) = \left(-\frac{5}{2}, \frac{y-2}{2}\right)$....(1)

For AC,

$(x, y) = \left(\frac{x-5}{2}, \frac{3+2}{2}\right)$

$(x, y) = \left(\frac{x-5}{2}, \frac{5}{2}\right)$....(2)

Equating 1 and 2, we get,

$$\frac{x-5}{2} = -\frac{5}{2}$$

x − 5 = - 5

x = 0

And,

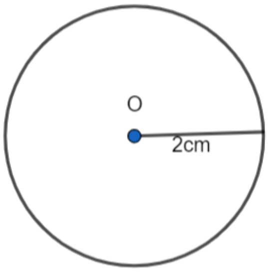
y − 2 = 5

y = 7

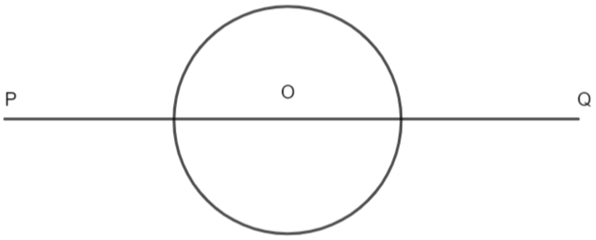
Hence, x = 0 and y = 7.

29. Solution: Steps of construction:

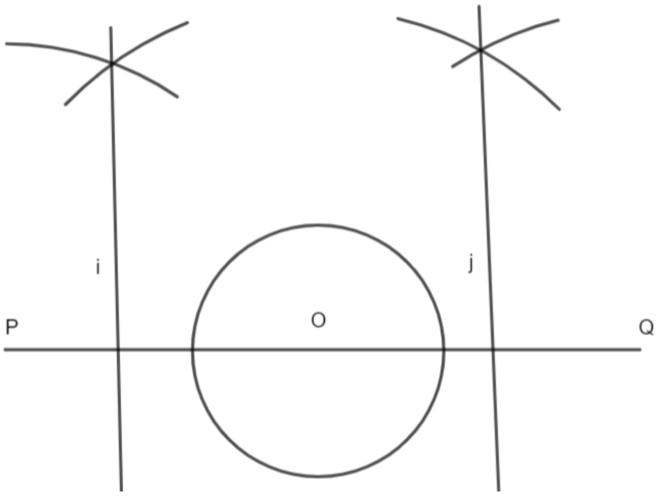
1. Draw a circle of radius 2cm with centre O.



2. Extend its diameter on both the sides and cut OP = OQ = 4cm.

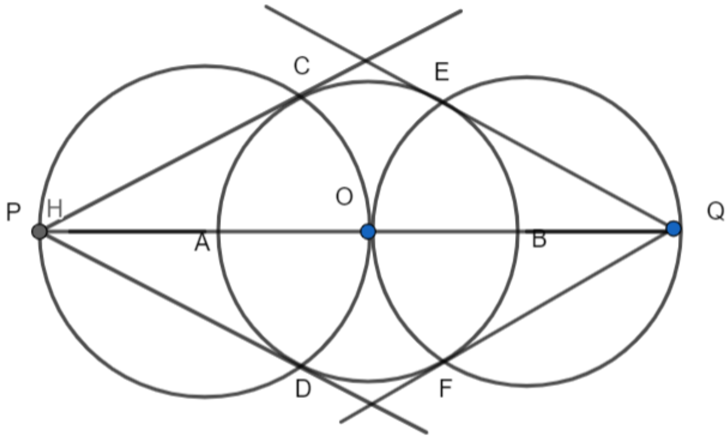


3. Now bisect OP and OQ. Let A and B be the points of OP and OQ respectively.



4. Taking A as the centre and OA as the radius draw a circle to intersect circle of radius 2cm at two points C and D. Now taking B as the centre and OB as the radius, draw a circle to intersect circle of radius 2cm at two points E and F.

5. Join PC, PD, QE and QF. These are the required tangents from P and Q to circle of radius 2cm.



30. Solution: Let $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$

Now, 1 and ½ are the zeroes of the polynomial. So, by factor theorem we know that $(x - 1)$ and $(x - ½)$ will completely divide $p(x)$

$(x - 1)(2x - 1)$ is a factor of $p(x)$

$2x^2 - 3x + 1$ is a factor of $p(x)$.

Dividing $p(x)$ by $2x^2 - 3x + 1$, we get,

$$
\begin{array}{r|l}
 & x^2 - 2 \\
\cline{2-2}
2x^2 - 3x + 1 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & 2x^4 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^2 + 6x - 2 \\
 & -4x^2 + 6x - 2 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Therefore,

$2x^4 - 3x^3 - 3x^2 + 6x - 2 = (2x^2 - 3x + 1)(x^2 - 2)$

Now, for finding the zeroes,

Putting $(2x^2 - 3x + 1)(x^2 - 2) = 0$

$x^2 - 2 = 0$

$(x + \sqrt{2})(x - \sqrt{2}) = 0$

$x = -\sqrt{2}$ or $x = \sqrt{2}$

Hence, the zeroes of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$ are 1, ½ , $\sqrt{2}$ and $-\sqrt{2}$.

31. Solution: Given: radius of the quadrant = 28cm

This will be the diagonal of the square.

Let the side of the square = acm

By Pythagoras theorem,

$a^2 + a^2 = 28^2$

$2a^2 = 784$

$a^2 = 392cm^2$

We know that, Area of square = $(side)^2 = a^2$

Therefore, Area of square = $392cm^2$

Now, Area of a quadrant of a circle = $\frac{1}{4} \pi r^2$

Putting r = 28cm, we get

Area of a quadrant of a circle $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

Area of a quadrant of a circle = (22×28) cm^2

Area of a quadrant of a circle = $616cm^2$

Area of shaded region = Area of a quadrant of the circle – Area of square

Area of shaded region = $(616 - 392)$ cm^2

Hence, the area of the shaded region is 224 cm^2

OR

Solution: Given: Length of the minute hand = 10cm

First, we need to calculate the angle swept by the minute hand from 6:10 to 7:05

In 60 minutes, minute hand sweep 360°.

Therefore, in 1 minute it will sweep $= \frac{360^\circ}{60} = 6^\circ$

Difference in minutes from 6:10 to 7:05 = 55 minutes

The angle swept by the minute hand = $55 \times 6^\circ$

Therefore, the angle swept by minute hand = 330°

We know that area of sector $= \frac{\theta}{360^\circ} \times \pi \times r^2$

Here, $\theta = 330^\circ$ and r = 10cm

Therefore,

Area swept by minute hand $= \left(\frac{330}{360}\right)^\circ \times \frac{22}{7} \times 10^2$

Area swept by minute hand $= \frac{11}{12} \times \frac{22}{7} \times 100$

Area swept by minute hand = 288.1 cm^2

32. Solution: Given Equation: $\frac{1}{x} - \frac{1}{x-3} = \frac{4}{3}$

Taking L.C.M, we get,

$$\frac{x-3-x}{x(x-3)} = \frac{4}{3}$$

$$-\frac{3}{x^2-3x} = \frac{4}{3}$$

Cross-multiplying, we get,

$-9 = 4(x^2 - 3x)$

$-9 = 4x^2 - 12x$

$4x^2 - 12x + 9 = 0$

Now, we know that for an equation $ax^2 + bx + c = 0$, roots are given by,

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Comparing $4x^2 - 12x + 9 = 0$ with $ax^2 + bx + c = 0$

a = 4, b = -12 and c = 9

Therefore,

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 4 \times 9}}{2 \times 4}$$

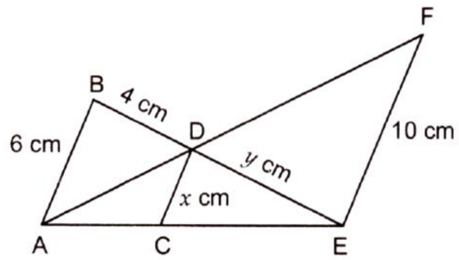
$$x = \frac{12 \pm \sqrt{144 - 144}}{8}$$

$$x = \frac{12}{8} = \frac{3}{2}$$

Hence, the roots of the equation are $\frac{3}{2}$ and $\frac{3}{2}$.

33. Solution: Theorem Used:

If two corresponding angles of two triangles are equal, the triangles are said to be similar.



Consider ΔADB and ΔEDF,

∠ADB = ∠EDF (Vertical angles)

∠ABD = ∠FED (Alternate angles)

∠BAD = ∠EFD (Alternate angles)

∴ ΔADB ~ ΔEDF (AAA criterion)

$\Rightarrow \frac{BD}{DE} = \frac{AB}{FE}$

$\Rightarrow \frac{4}{y} = \frac{6}{10}$

⇒ 6y = 40

$\Rightarrow y = \frac{40}{6} = \frac{20}{3}$

⇒ y = 6.67

Similarly, ΔADE ~ ΔCDE (AA criterion)

$\Rightarrow \frac{DE}{BE} = \frac{DC}{BA}$

$\Rightarrow \frac{6.67}{10.67} = \frac{x}{6}$

$\Rightarrow x = 6 \times \left(\frac{6.67}{10.67}\right)$

⇒ x = 3.75

Hence, the values of x and y are 3.75 and 6.67.

Part –B

34. Solution: Given: Height of the vessel = diameter of the cylindrical vessel

∴ h = 2r

Given: water filled in the cylindrical vessel is poured in two cylindrical vessels

Volume of a cylindrical vessel = π r^2 h

So, volume = $\pi r^2(2r) = 2\pi r^3$

Volume of vessel with diameter 42 cm and height 21 cm

$= \pi \times \left(\frac{42}{2}\right)^2 \times 21$

$= \pi (21)^3 \text{ cm}^3$

Since cylinders are filled by water completely, so volume of large cylinder = sum of these small cylinders
$\therefore 2\pi r^3 = 2 \times \pi (21)^3$

$\Rightarrow r = 21\text{cm}$

$\therefore$ diameter of the cylindrical vessel = 42cm

35. Solution:

Class interval (exclusive)	Class interval (inclusive)	Class interval (Frequency)	Cumulative frequency
160-162	159.5-162.5	15	15
163-164	162.5-165.5	118	133 (F)
166-168	165.5-168.5	142 (f)	275
169-171	168.5-171.5	127	402
172-174	171.5-174.5	18	420
		N = 420	

We have, N = 420

$\frac{N}{2} = \frac{420}{2} = 210$

The cumulative frequency just greater than $\frac{N}{2}$ is 275 then 165.5-168.5 is the median class such that,

l = 165.5, f = 142, F = 133 and h = 168.5-165.5 = 3

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

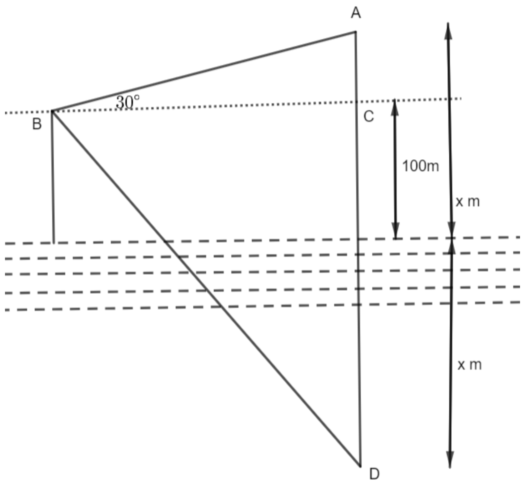
= 165.5 + $\frac{210-133}{142}$ * 3

= 165.5 + $\frac{77}{142}$ * 3

= 165.5 + 1.63

= 167.13

36. Solution:



A is the helicopter and D is its reflection in the lake.
Let the height of the helicopter be x. and BC = y.
In ΔABC,

$$tan30° = \frac{AC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{x - 100}{y}$$

y = √3(x – 100)m.....(1)
In ΔBCD,

$$tan45° = \frac{CD}{BC}$$

$$1 = \frac{x + 100}{y}$$

y = (x + 100)m…….(2)
From equations 1 and 2,
x + 100 = √3(x – 100)
x + 100 = √3x - 100√3
√3x – x = 100 + 100√3
x(√3 – 1) = 100(1 + √3)

$$x = \frac{100\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$$

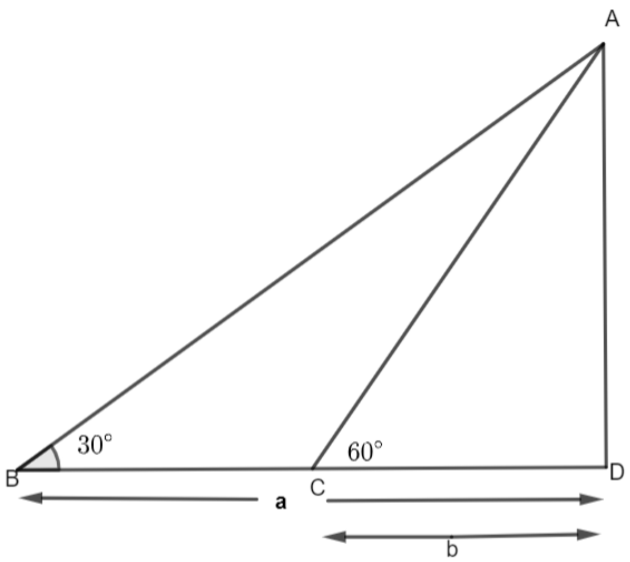
Rationalizing the fraction, we get

$$x = \frac{100\left(\sqrt{3}+1\right)^2}{2}m$$

$$x = 50\left(4+\sqrt{3}\right)m$$

OR

Solution:



Let the height of the tower, AD = h m
Now, in ΔADC, we have,

$$tan60° = \frac{AD}{CD}$$

$$\sqrt{3} = \frac{h}{b}$$

h = b√3m

$$b = \frac{h}{\sqrt{3}} \ldots(1)$$

Similarly, in ΔADB, we have,

$$tan30° = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{a - b}$$

h√3 = a − b

Putting the value of b from equation 1, we get,

$$h\sqrt{3} = a - \frac{h}{\sqrt{3}}$$

$$h\sqrt{3} + \frac{h}{\sqrt{3}} = a$$

$$\left(\frac{4h}{\sqrt{3}}\right) = a$$

$h = \frac{\sqrt{3}a}{4} m$.....(2)

From (1) and (2)

$$\frac{\sqrt{3}a}{4} = \sqrt{3}b$$

$$\frac{a}{b} = 4$$

$$\sqrt{\frac{a}{b}} = 2$$

Hence, the value of $\sqrt{\frac{a}{b}}$ is 4.

***